Stellar astrophysics and optical interferometry

Denis Mourard – UCA/OCA/CNRS Lagrange
Overall schedule of the lecture

Monday
• Principle and reality of optical interferometry
• History of the developments

Tuesday
• The modern developments
• The main science programs

Wednesday
• Practical session
• Use of the preparation and analysis tools

Thursday
• Future developments
Steps towards an interferometric observation

- Got an idea for possible high angular resolution views on your favourite object
  - See examples
  - Read papers
  - Think to your favourite object as a brightness distribution on the sky

- Evaluate the feasibility
  - Use ASPRO2 software and select your interferometer/instrument/mode

- You got time, congratulations!
  - Prepare the actual observations, select calibrators with SearchCal

- You obtain data
  - Use the model fitting tools LitPRO
THE EFFECTS OF LIMB DARKENING ON MEASUREMENTS OF ANGULAR SIZE WITH AN INTENSITY INTERFEROMETER

R. Hambury Brown, J. Davis, R. J. W. Lake and R. J. Thompson

2. A SIMPLE REPRESENTATION OF LIMB DARKENING

In the conventional linear representation of limb darkening the distribution of brightness across the star’s disc is given by

$$I_\lambda(\mu) = I_\lambda(1)(1-u_\lambda(1-\mu))$$

where $I_\lambda(\mu)$ is the brightness of a point on the disc at a wavelength $\lambda$, $\mu$ is the cosine of the angle between the normal to the surface at that point and the line of sight from the star to the observer, and $u_\lambda$ is the limb-darkening coefficient. By taking the Hankel transform of the apparent angular distribution of intensity across the source it can be shown that,

$$\Gamma_\lambda^{2}(d) = (\alpha/2+\beta/3)^{-2}[\alpha J_3(\alpha)/\alpha+\beta(\pi/2)^{-2} J_3/3(\alpha)/\alpha^{3/2}]^2$$

where $\alpha = 1-u_\lambda$, $\beta = u_\lambda$, $x = \pi \theta_{LD} d/\lambda_0$, $\theta_{LD}$ is the true angular diameter of the limb-darkened star, and it is assumed that $\Delta_\lambda = 1$.

FIG. 3. The variation of correlation $\Delta_\lambda \Gamma_\lambda^{2}(d)$ with baseline $d$ for Sirius. The points show the observed values; the full line is a theoretical curve, based on a model stellar atmosphere ($T_e = 10000\text{K}$, $\log g = 4$), with zero-baseline correlation and angular size adjusted to give the best fit to the observations. The broken lines represent the rms uncertainty in the theoretical curve.
Limb darkening & interferometry - “classical considerations”

Limb-darkening corrections for interferometric uniform disc stellar angular diameters

J. Davis, W. J. Tango and A. J. Booth†


\[ \rho_\theta = \theta_{\text{LD}} / \theta_{\text{UD}} \]

Measure of $\theta_{\text{UD}}$ + estimation of $u$ the linear limb darkening coefficient (Te, logg) $\rightarrow \rho_\theta = \sqrt{\frac{1-u}{\frac{3}{7}u}} \rightarrow \theta_{\text{LD}}$

(a): Te=3500K, log=4.5
(b): Te=5500K, log=4.5
(c): Te=10000K, log=4.0
(d): Te=15000K, log=4.0
(e): Te=20000K, log=4.0
Different definitions of Limb Darkening

From Kervella et al., A&A 597 (2017)

- uniform disk:
  \[ I(\mu)/I(1) = 1; \]  \hspace{1cm} (2)

- linear:
  \[ I(\mu)/I(1) = 1 - u (1 - \mu); \] \hspace{1cm} (3)

- power law (Hestroffer 1997):
  \[ I(\mu)/I(1) = \mu^a; \] \hspace{1cm} (4)

- quadratic:
  \[ I(\mu)/I(1) = 1 - a (1 - \mu) - b (1 - \mu)^2; \] \hspace{1cm} (5)

- square root:
  \[ I(\mu)/I(1) = 1 - c (1 - \mu) - d (1 - \sqrt{\mu}); \] \hspace{1cm} (6)

- four-parameter:
  \[ I(\mu)/I(1) = 1 - \sum_{k=1}^{4} a_k (1 - \mu^{k/2}). \] \hspace{1cm} (7)

In addition, we consider the following polynomial model with six parameters:

- polynomial:
  \[ I(\mu)/I(1) = \frac{\sum_{k=0}^{5} s_k \mu^k}{\sum_{k=0}^{5} s_k}. \] \hspace{1cm} (8)

**Fig. 2.** Comparison of different parametric limb darkening models of the Sun with the observed limb darkening profile measured by Pierce et al. (1977) in the H band. The residuals in percentage of the observed intensity profile are shown in the lower panel.
Actual measurements with VLTI/PIONIER

Kervella et al. A&A 2017
\[ \theta_{LD} = 8.5 \text{mas}, \ u = 0.24 \]

![Graph showing adjustment of a power law limb darkened disk model to the PIONIER squared visibilities of \( \alpha \) Cen A (solid gray curve). The dashed gray curve represents the best-fit uniform disk model. The bottom panels show the residuals of the fit in number of times the statistical error bar. The coverage of the (\( u, v \)) plane is shown in the upper right corner.]

Fig. 3. Adjustment of a power law limb darkened disk model to the PIONIER squared visibilities of \( \alpha \) Cen A (solid gray curve). The dashed gray curve represents the best-fit uniform disk model. The bottom panels show the residuals of the fit in number of times the statistical error bar. The coverage of the (\( u, v \)) plane is shown in the upper right corner.
Activities

First exercise:
- Reproduce the preparation for α Cen A (VLTI PIONIER, θLD=8.5mas, u=0.24)

Second exercise
- Can I measure the limb darkened diameter of a G0V star @ 20pc?
- Try with PIONIER on the previous ASPRO2 file (change the diameter of α Cen A)
- Example of HD84737. θLD=[0.7-0.9], u=0.5
- Use CHARA/Future/SPICA in ASPRO2 and generates simulated data
- Test with LitPRO software
- Conclusion
- Can I find good calibrators for HD84737? Use Searchcal

More activities with LitPRO with actual data (JMMC VLTI school 2017, Isabelle & Michel Tallon)
- Ex. 3.1/3.2: Arcturus at one wavelength
- Ex. 3.3: Arcturus at two wavelengths with linked parameters
- Ex. 4: ThetaOriC: binary star. 1 observation, 2 observations
Exercise 3.1- simple fit

• Load the file **arcturus.1.79mu.oifits**
  What kind of data did you load ?

• Explore the data
  – Settings Tree ➔ **Files**
    in Files Panel, Plot VIS2DATA..., UV coverage,...
    What clue do you get from the OI_T3 data ?

• Build a model
  – Settings Tree ➔ **Targets**
  – Add new target
  – In Target Panel / Model List, add a model function (for ex. disk)
  – Initialize the parameters and select the data to fit (for ex. VIS2)

• **Run fit** (bottom of the Settings tree)

• Visualize the result of the fit:
  – Settings Tree ➔ **Results** (and personal notebook)
  – with plots from Plot model Panel : Plot Radial (ex: VIS2, try "Residuals", "overplot model")

• Try a fit after removing the setting “Normalize total flux”:
  Explain the value of the flux_weight parameter and of Chi2
Exercise 3.1 - simple fit

arcturus.1.79mu.oifits + disk

Good fit

reduced Chi2: initial= 3.746e+04 -
final= 0.4392 - sigma= 0.2828
Number of degrees of freedom = 25
Number of iterations: 9 (max number= 200)

Final values and standard deviation for fitted parameters:
diameter1 = 20.348 +/- 0.0177 mas

Concerning the normalization:
- Flux is no constrained by interferometric data
- Either:
  - o fix the flux
  - o use Normalize total flux (add a constrain to chi2)
Exercise 3.2- simple fit (second wavelength)

- Open a *New setting* and load the file *arcturus.1.52mu.oifits*

- Process as Exercise 1.1
  ex: same model function *disk*

- *Run fit* from various initial values of the diameter (*value* = 0 mas, 20 mas, 25 mas)
  How are the results of these fits?

- Explore the “Chi2 space” for analysis:
  
  **Plot Chi2 1D** with Parameter[diameter1] (*log & reduced* selected) (min= 0, max= 30, #samples=100)

  What do you observe?
  
  Why the final Chi2 is not so good when fitting from the global minimum?

- Try another model: a center to limb-darkening model, for ex. *limb_power*
**Exercise 3.2**

arcturus.1.52mu.oifits + disk

**Bad fit**

Reduced Chi2: initial = 3.588e+07 - final = 1.043e+04

\[ \sigma = 0.267 \]

\[ \text{diameter}_1 = 13.694 \pm 0.445 \text{ mas} \]

**Good fit?**

Reduced Chi2: initial = 1311 - final = 11.87

\[ \sigma = 0.267 \]

\[ \text{diameter}_1 = 20.333 \pm 0.0398 \text{ mas} \]

No!

And have a look on the residuals of the fit…

"with initial diameter value = 0"

"with initial diameter value = 20 mas"
Exercise 3.3 - Fit with sharing of parameter

• **Aim**: on 2 data sets, one by wavelength, fit a model of center-to-limb darkening (e.g. power law) considering that:
  – the diameter of the photosphere (therefore common to both groups) is achromatic
  – the center-to-limb darkening coefficient is chromatic

• Open a New setting and load the files *arcturus.1.52mu.oifits* and *arcturus.1.79mu.oifits*
  – *Add new target* for file arcturus.1.52mu.oifits and select *limb_power* ---*group1*
  – *Add new target* for file arcturus.1.79mu.oifits and select *limb_power* ---*group2*

• Share the diameter between both groups, using contextual menu (mouse right click) in the Parameters table:
  – for *diameter1*: *share this parameter*
  – for *diameter2*: *link it with diameter1*
  – you may verify with *Shared parameters* of the Settings tree

• *Run fit*

• Plot all the data and fitted models on the **same** graph:
  – use the *Common plots panel* accessible when clicking on Settings Tree → *Plots*
  – select both targets
Exercise 3.3 - Fit with sharing of parameter

\[
\text{diameter}_1 = 20.68 \pm 0.02 \text{ mas} \\
\text{power}_1 = 0.229 \pm 0.012 \text{ @1.52 mm} \\
\text{power}_2 = 0.118 \pm 0.0115 \text{ @1.79 mm}
\]

Initial reduced Chi2 = 258.8  -  Final reduced Chi2 = 0.7491
Exercise 4 - Fit with degeneracies

- **Aim:** estimate the separation of a binary
- Open a New setting and load the file `Theta1Ori2007Dec03_2.fits`
- Build the model:
  - combine 2 puncts
  - select VIS2 only
  - leave the parameters $x_1, y_1$ fixed at 0. (it is the default case)
  - bound the flux_weight to [0,1] and take for ex. 0.5 as initial value
  - run fit
    Look at $x_2, y_2$ values
- Bound the parameters $x_2, y_2$ between -30 and 30 mas and run fit again
  Look at the errors bars on $x_2, y_2$ and explain what happens
- Use **Plot Chi2 2D** with parameters ($x_2, y_2$)
- Observe the chi2 map
  - explain the valleys and their orientation
  - explain the correlation between $x_2$ $y_2$ and the symmetry of the Chi2 map

- So, what to do for resolving the binary Theta1 Ori C?
Exercise 4 - Fit with degeneracies

Theta1Ori2007Dec03_2.fits

Plot Chi2 2D, $x_2$, $y_2$

Infinity of $[x_2, y_2]$ giving the same Chi2 minimum

$\Rightarrow$ no single solution = degeneracy

$[x_2, y_2]$ are correlated

Example of a result of a fit:

$\text{flux\_weight1} = 0.73235 +/- 0.395$
$\text{flux\_weight2} = 0.26765 +/- 0.145$
$x_2 = -9.7938 +/- 3.7e+04 \text{ mas}$
$y_2 = -29.335 +/- 1.24e+05 \text{ mas}$

reduced Chi2 final = 23.59 - sigma = 0.161165

--- Correlation matrix ---

$\begin{bmatrix}
i1 & i2 & x2 & y2 \\
i1 & 1 & 0.99 & -0.062 & -0.062 \\
i2 & 0.99 & 1 & -0.023 & 0.023 \\
x2 & 0.062 & -0.023 & 1 & -1 \\
y2 & -0.062 & 0.023 & -1 & 1
\end{bmatrix}$

uv coverage
Exercise 4 - Fit with degeneracies

- In the current setting, add the file `Theta1Ori2007Dec05_2.fits` and select it.
- Use again `plot chi2 2D` with `(x2, y2)`
  - set the `(x2, y2)` corresponding to the best minimum of `chi2`, as well as both `flux_weights` and fit the `VIS2` only.
- Find the two best solutions and compare them.

Are they different?

See the possibility to convert `(x2, y2)` in polar coordinates: use contextual menu on `x2` or `y2` in the Parameters table.

This model is located at `sep=18.719mas' PA=60.942deg` relatively to the center of this target. or `rho=18.72 PA=240.942`
Exercise 4 - Fit with degeneracies

Plot Chi2 2D, $x_2$, $y_2$

Theta1Ori2007Dec03_2.fits + Theta1Ori2007Dec05_2.fits

2 equivalent solutions

flux_weight1 = 0.69414 +/- 0.49
flux_weight2 = 0.30586 +/- 0.216
$x_2$ = 16.364 +/- 0.047 mas or -16.364
$y_2$ = 9.092 +/- 0.0709 mas or -9.092

reduced Chi2 final = 81.53

i.e. [$\rho$, PA] = [18.72mas, 60.94°] or [18.72mas, 240.94°]
Exercise 4 - Fit with degeneracies

• On the same setting, add the T3phi for the fit
• *run fit* from one of the best set of fitted parameters
• *run fit* from the second best set of fitted parameters
• Conclusion?
• Compare your result with the published one: Kraus S. et al, 2009, A&A. 497-1, pp. 195-207

<table>
<thead>
<tr>
<th>Date</th>
<th>Filter</th>
<th>Flux ratio</th>
<th>PA(°)</th>
<th>ρ (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VLT/AMBER</td>
<td>2007.9233</td>
<td>H+K</td>
<td>0.24±0.07</td>
<td>241.2±1</td>
</tr>
</tbody>
</table>

• Conclusion?
• **Plot Radial VIS2**
  - infer how to improve the model
  - you may check your answer with a teacher
• You may also plot an image of your model:
Exercise 4 - Fit with degeneracies

Only one best solution:

- flux_weight1 = 0.6975 +/- 0.37
- flux_weight2 = 0.3025 +/- 0.16
- $x_2 = -16.372 +/- 0.0406$ mas
- $y_2 = -9.0853 +/- 0.0613$ mas

i.e. $[\rho, PA] = [18.72$ mas, $241^\circ]$

- flux_ratio = 0.43
- reduced Chi2 final = 62.29

Published results:


<table>
<thead>
<tr>
<th>Date</th>
<th>Filter</th>
<th>Flux ratio</th>
<th>PA ($^\circ$)</th>
<th>$\rho$ (mas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VLT/AMBER</td>
<td>2007.9233</td>
<td>$H+K$</td>
<td>$0.24 \pm 0.07$</td>
<td>$241.2 \pm 1$</td>
</tr>
</tbody>
</table>

$\pi$ dephasing for the bad solution
Exercise 4 - Fit with degeneracies

Theta1Ori2007Dec03_2.fits + Theta1Ori2007Dec05_2.fits

VIS2 + T3Phi

Decreasing of VIS2 with the spatial frequency

2 puncts... but the interferometer may resolve the components of the binary since resolved components give decreasing VIS2.
Exercise 4 - Fit with degeneracies

Theta1Ori2007Dec03_2.fits  +  Theta1Ori2007Dec05_2.fits  
VIS2 + T3Phi

Fit with 2 disks:

diameter1 = 2.0039  +/- 0.055  mas

diameter2 = 2.2654  +/- 0.197  mas

Better T3phi fit

Better compatibility for the flux ratio with the published result (0.24)